Welcome to AP Calculus!

I'm so excited to work with all of you starting in the Fall in AP Calculus! It is a fast-moving course with an obvious immovable deadline – the AP Test in May!

As a result, summer work to dust off your Algebra skills (and a little bit of Trig) is necessary.

Here is some important information about this summer work packet. A printed copy will be provided - this online copy is backup in case the printed copy is lost or damaged.

1) The packet's format is a page or two of review followed by a worksheet, followed by another review page, and another worksheet and so on. So only about half the packet is worksheets. **You need to use the review pages to dust off a skill. Pre-calculus doesn't repeat a lot of Algebra 2 so there are things here you haven't seen in well over a year.**
2) You may begin the first part (Section R and Speed Trig) at any time.
3) I prefer the rest of the packet (beginning with Part A) be completed starting July 1, so it is fresh in your memory. If you must do it earlier, be sure to review before school starts!
4) Some parts are intentionally left out, so do not be alarmed.
5) Beginning July 1, you will find helpful resources for this packet at the website: https://haberlandt.weebly.com. This will include tutorial videos.
6) We will have a test (closed note) on the content on the 2nd full meeting of class. Most test questions will be completed without a graphing calculator, which is true for most of the packet also.
7) On the test, you are responsible for all skills in the packet.
8) You will also earn completion points on the packet. You will receive full completion points if you complete 70% or more of the Assignment problems in the packet. <u>This allows you to skip problems that seem too easy</u>. You must show your work for problems that require it to receive credit for them. Partial completion points will be awarded for less than 70% completion. You can write problems on notebook paper, squeeze work on the worksheet pages, or work electronically on an Ipad or other device. <u>I will collect your work the day of the test.</u>
9) Answers are included in a separate answer document. If you are finding this packet online, email Mrs. H at rhaberlandt@fenwickfalcons.org to receive the answer document.
10) I encourage you to form study groups and check your Fenwick email often in the summer for helpful tips. It is NOT cheating to work with a study group or ask another student for help with a problem. Of course, copying someone else's work is a violation of academic integrity policies (and obviously does not result in you dusting off the skills!).
11) Since this is a college level course I will not cc parents on any emails unless I have concerns about your progress. If you do not do well on the test on this review material, then I will reach out to you and your parents and discuss your effort on the summer review assignment and appropriate next steps.
12) If you have questions on the assignment, please email me a picture of your work and I'd be glad to help. I will also go over questions in the first class but it is highly recommended that you ask questions this summer so you can keep moving forward.
13) If a topic seems completely foreign to you, meaning that it wasn't covered in Honors Algebra 2 or Honors Pre-Calc, please let me know.

AP Calculus is a challenging and fun journey. Let's BEGIN!!!
Mrs. Haberlandt

R. Special Angles Begin any time - Section R and Speed Trig

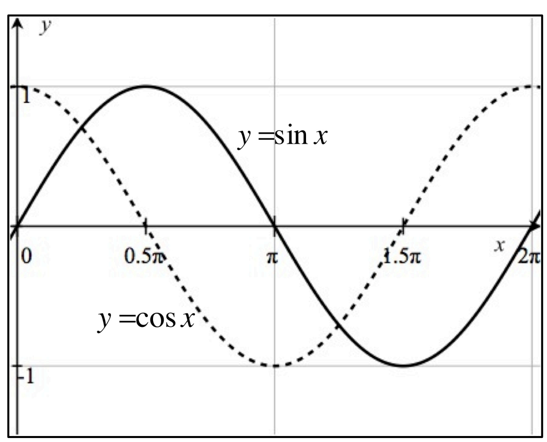
Students must be able to find trig functions of quadrant angles $\left(0,\ 90°, 180°, 270°\right)$ and special angles, those based on the $30° - 60° - 90°$ and $45° - 45° - 90°$ triangles.

First, for most calculus problems, angles are given and found in radians. Students must know how to convert degrees to radians and vice-versa. The relationship is 2π radians = $360°$ or π radians = $180°$. Angles are assumed to be in radians so when an angle of $\frac{\pi}{3}$ is given, it is in radians. However, a student should be able to picture this angle as $\frac{180°}{3} = 60°$. It may be easier to think of angles in degrees than radians, but realize that unless specified, angle measurement must be written in radians. For instance, $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$.

The trig functions of **quadrant angles** $\left(0,\ 90°, 180°, 270° \text{ or } 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\right)$ can quickly be found. Choose a point along the angle and realize that *r* is the distance from the origin to that point and always positive. Then use the definitions of the trig functions.

θ	point	x	y	r	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
0	$(1,0)$	1	0	1	0	1	0	does not exist	1	does not exist
$\frac{\pi}{2}$ or $90°$	$(0,1)$	0	1	1	1	0	does not exist	1	does not exist	0
π or $180°$	$(-1,0)$	-1	0	1	0	-1	0	does not exist	-1	does not exist
$\frac{3\pi}{2}$ or $270°$	$(0,-1)$	0	-1	1	-1	0	Does not exist	-1	does not exist	0

If you picture the graphs of $y = \sin x$ and $y = \cos x$ as shown to the right, you need not memorize the table. You must know these graphs backwards and forwards.



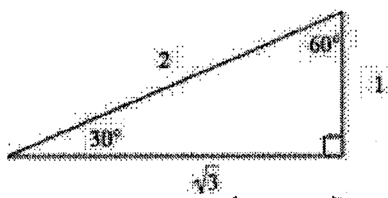
• Without looking at the table, find the value of

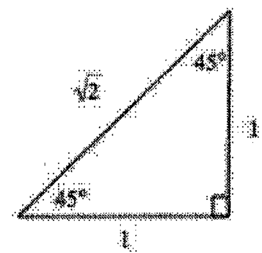
a. $5\cos 180° - 4\sin 270°$

$$\boxed{\begin{array}{l} 5(-1) - 4(-1) \\ -5 + 4 = -1 \end{array}}$$

b. $\left(\dfrac{8\sin\frac{\pi}{2} - 6\tan\pi}{5\sec\pi - \csc\frac{3\pi}{2}}\right)^2$ $\boxed{\left[\dfrac{8(1) - 6(0)}{5(-1) - (-1)}\right]^2 = \left(\dfrac{8}{-4}\right)^2 = 4}$

Because over half of the AP exam does not use a calculator, you must be able to determine trig functions of **special angles.** You must know the relationship of sides in both $30°-60°-90°\left(\frac{\pi}{6},\frac{\pi}{3},\frac{\pi}{2}\right)$ and $45°-45°-90°\left(\frac{\pi}{4},\frac{\pi}{4},\frac{\pi}{2}\right)$ triangles.

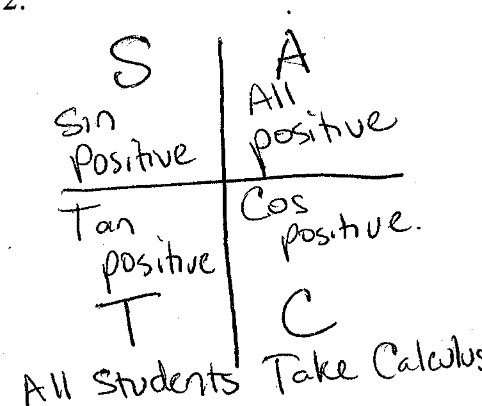




In a $30°-60°-90°\left(\frac{\pi}{6},\frac{\pi}{3},\frac{\pi}{2}\right)$ triangle, the ratio of sides is $1-\sqrt{3}-2$.

In a $45°-45°-90°\left(\frac{\pi}{4},\frac{\pi}{4},\frac{\pi}{2}\right)$ triangle, the ratio of sides is $1-1-\sqrt{2}$.

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
$30°\left(\text{or } \frac{\pi}{6}\right)$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$45°\left(\text{or } \frac{\pi}{4}\right)$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$60°\left(\text{or } \frac{\pi}{3}\right)$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$



Special angles are any multiple of $30°\left(\frac{\pi}{6}\right)$ or $45°\left(\frac{\pi}{2}\right)$. To find trig functions of any of these angles, draw them and find the **reference angle** (the angle created with the x-axis). Although most problems in calculus will use radians, you might think easier using degrees. This will create one of the triangles above and trig functions can be found, remembering to include the sign based on the quadrant of the angle. Finally, if an angle is outside the range of $0°$ to $360°(0 \text{ to } 2\pi)$, you can always add or subtract $360°(2\pi)$ to find trig functions of that angle. These angles are called **co-terminal angles.** It should be pointed out that $390° \neq 30°$ but $\sin 390° = \sin 30°$.

• Find the exact value of the following

a. $4\sin 120° - 8\cos 570°$

Subtract 360° from 570°
$4\sin 120° - 8\cos 210°$
120° is in quadrant II with reference angle 60°.
210° is in quadrant III with reference angle 30°.
$4\left(\frac{\sqrt{3}}{2}\right)-8\left(\frac{-\sqrt{3}}{2}\right)=6\sqrt{3}$

b. $\left(2\cos\pi - 5\tan\frac{7\pi}{4}\right)^2$

$(2\cos 180° - 5\tan 315°)^2$
180° is a quadrant angle
315° is in quadrant III with reference angle 45°
$[2(-1)-5(-1)]^2 = 9$

SPEED TRIG - NEW

No CALCULATOR!

Date________________ Block____

Find the exact value of each trigonometric function.

1) $\cos \frac{4\pi}{3}$

2) $\cos \frac{11\pi}{6}$

3) $\cos \frac{7\pi}{6}$

4) $\cos 0$

5) $\tan \frac{\pi}{3}$

6) $\cos \frac{\pi}{2}$

7) $\sin \frac{3\pi}{2}$

8) $\sin \frac{11\pi}{6}$

9) $\cos \frac{5\pi}{4}$

10) $\tan 0$

11) $\cos \frac{3\pi}{2}$

12) $\cos \frac{5\pi}{3}$

13) $\sin \frac{\pi}{2}$

14) $\sin \frac{\pi}{3}$

15) $\tan \frac{7\pi}{4}$

16) $\cos \frac{3\pi}{4}$

17) $\tan \frac{\pi}{6}$

18) $\sin \frac{5\pi}{3}$

19) $\tan \frac{2\pi}{3}$

20) $\tan \frac{5\pi}{4}$

21) $\tan \frac{3\pi}{4}$

22) $\cos \frac{\pi}{3}$

23) $\tan \frac{7\pi}{6}$

24) $\tan \frac{4\pi}{3}$

25) $\sin \frac{5\pi}{4}$

26) $\tan \frac{11\pi}{6}$

27) $\cos \frac{7\pi}{4}$

28) $\sin \frac{4\pi}{3}$

29) $\tan \frac{5\pi}{6}$

30) $\cos \frac{2\pi}{3}$

31) $\sin \frac{7\pi}{4}$

32) $\sin 0$

33) $\tan \frac{\pi}{2}$

34) $\tan \frac{\pi}{4}$

35) $\sin \frac{\pi}{6}$

36) $\cos \frac{\pi}{6}$

37) $\sin \frac{7\pi}{6}$

38) $\sin \frac{3\pi}{4}$

39) $\cos \pi$

40) $\sin \frac{\pi}{4}$

41) $\sin \pi$

42) $\sin \frac{2\pi}{3}$

43) $\cos \frac{\pi}{4}$

44) $\cos \frac{5\pi}{6}$

45) $\tan \pi$

46) $\tan \frac{3\pi}{2}$

47) $\tan \frac{5\pi}{3}$

48) $\sin \frac{5\pi}{6}$

A. Functions

Remainder of packet - prefer you begin around July 1.

The lifeblood of precalculus is functions. A **function** is a set of points (x, y) such that for every x, there is one and only one y. In short, in a function, the x-values cannot repeat while the y-values can. In AB Calculus, all of your graphs will come from functions.

The notation for functions is either "$y =$" or "$f(x) =$". In the $f(x)$ notation, we are stating a rule to find y given a value of x.

1. If $f(x) = x^2 - 5x + 8$, find a) $f(-6)$ b) $f\left(\frac{3}{2}\right)$ c) $\frac{f(x+h) - f(x)}{h}$

a) $f(-6) = (-6)^2 - 5(-6) + 8$
$36 + 30 + 8$
74

b) $f\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^2 - 5\left(\frac{3}{2}\right) + 8$
$\frac{9}{4} - \frac{15}{2} + 8$
$\frac{11}{4}$

c) $\frac{f(x+h) - f(x)}{h}$
$\frac{(x+h)^2 - 5(x+h) + 8 - (x^2 - 5x + 8)}{h}$
$\frac{x^2 + 2xh + h^2 - 5x - 5h + 8 - x^2 + 5x - 8}{5}$
$\frac{h^2 + 2xh - 5h}{h} = \frac{h(h + 2x - 5)}{h} = h + 2x - 5$

Functions do not always use the variable x. In calculus, other variables are used liberally.

2. If $A(r) = \pi r^2$, find a) $A(3)$ b) $A(2s)$ c) $A(r+1) - A(r)$

$A(3) = 9\pi$

$A(2s) = \pi(2s)^2 = 4\pi s^2$

$A(r+1) - A(r) = \pi(r+1)^2 - \pi r^2$
$\pi(2r + 1)$

One concept that comes up in AP calculus is **composition of functions**. The format of a composition of functions is: plug a value into one function, determine an answer, and plug that answer into a second function.

3. If $f(x) = x^2 - x + 1$ and $g(x) = 2x - 1$, a) find $f(g(-1))$ b) find $g(f(-1))$ c) show that $f(g(x)) \neq g(f(x))$

$g(-1) = 2(-1) - 1 = -3$
$f(-3) = 9 + 3 + 1 = 13$

$f(-1) = 1 + 1 + 1 = 3$
$g(3) = 2(3) - 1 = 5$

$f(g(x)) = f(2x - 1) = (2x - 1)^2 - (2x - 1) + 1$
$= 4x^2 - 4x + 1 - 2x + 1 + 1 = 4x^2 - 6x + 3$
$g(f(x)) = g(x^2 - x + 1) = 2(x^2 - x + 1) - 1$
$= 2x^2 - 2x + 1$

Finally, expect to use **piecewise functions**. A piecewise function gives different rules, based on the value of x.

4. If $f(x) = \begin{cases} x^2 - 3, & x \geq 0 \\ 2x + 1, & x < 0 \end{cases}$, find a) $f(5)$ b) $f(2) - f(-1)$ c) $f(f(1))$

$f(5) = 25 - 3 = 22$

$f(2) - f(-1) = 1 - (-1) = 2$

$f(1) = -2,\ f(-2) = -3$

A. Function Assignment

• If $f(x) = 4x - x^2$, find

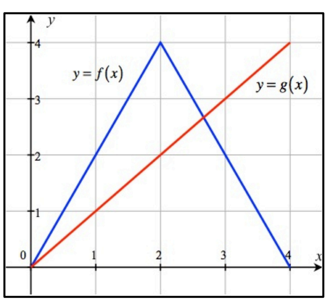
1. $f(4) - f(-4)$
2. $\sqrt{f\left(\frac{3}{2}\right)}$
3. $\dfrac{f(x+h) - f(x-h)}{2h}$

• If $V(r) = \frac{4}{3}\pi r^3$, find

4. $V\left(\frac{3}{4}\right)$
5. $V(r+1) - V(r-1)$
6. $\dfrac{V(2r)}{V(r)}$

• If $f(x)$ and $g(x)$ are given in the graph below, find

7. $(f - g)(3)$
8. $f(g(3))$



• If $f(x) = x^2 - 5x + 3$ and $g(x) = 1 - 2x$, find

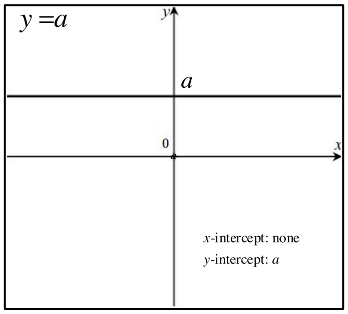
9. $f(g(x))$

• If $f(x) = \begin{cases} \sqrt{x+2} - 2, & x \ge 2 \\ x^2 - 1, & 0 \le x < 2 \\ -x, & x < 0 \end{cases}$, find

10. $f(0) - f(2)$
11. $\sqrt{5 - f(-4)}$
12. $f(f(3))$

C. Graphs of Common Functions

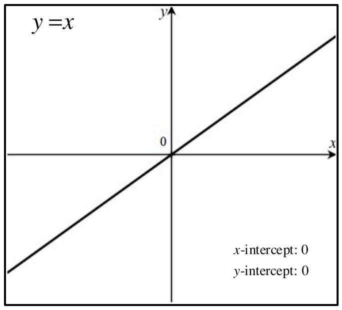
There are certain graphs that occur all the time in calculus and students should know the general shape of them, where they hit the x-axis (zeros) and y-axis (y-intercept), as well as the domain and range. There are no assignment problems for this section other than students memorizing the shape of all of these functions. In section 5, we will talk about transforming these graphs.



Function: $y = a$

Domain: $(-\infty, \infty)$

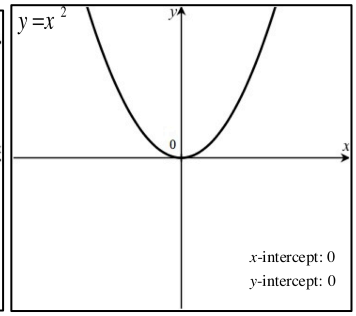
Range: $[a, a]$



Function: $y = x$

Domain: $(-\infty, \infty)$

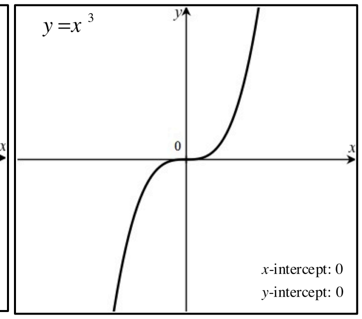
Range: $(-\infty, \infty)$



Function: $y = x^2$

Domain: $(-\infty, \infty)$

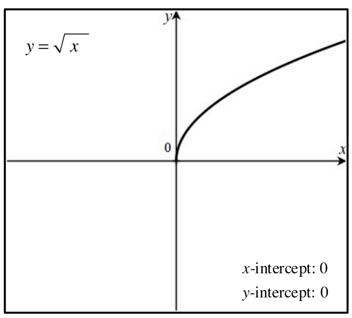
Range: $[0, \infty)$



Function: $y = x^3$

Domain: $(-\infty, \infty)$

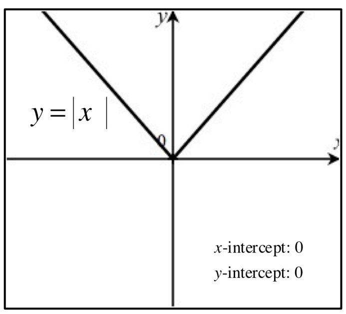
Range: $(-\infty, \infty)$



Function: $y = \sqrt{x}$

Domain: $[0, \infty)$

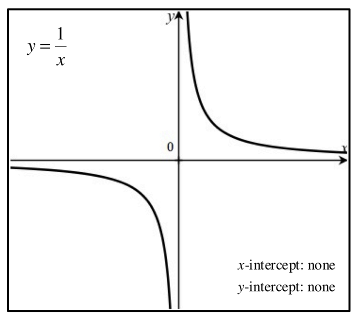
Range: $[0, \infty)$



Function: $y = |x|$

Domain: $(-\infty, \infty)$

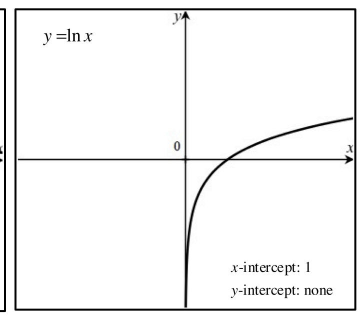
Range: $[0, \infty)$



Function: $y = \dfrac{1}{x}$

Domain: $x \neq 0$

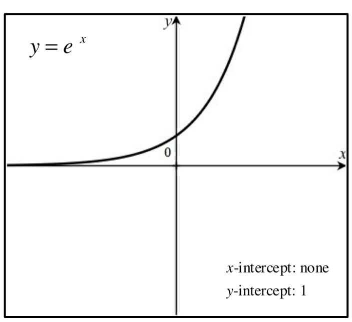
Range: $y \neq 0$



Function: $y = \ln x$

Domain: $(0, \infty)$

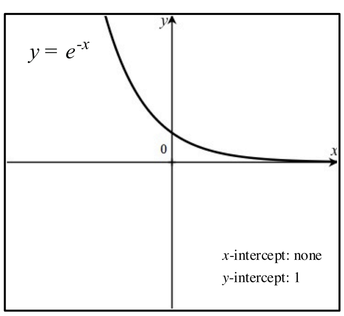
Range: $(-\infty, \infty)$



Function: $y = e^x$

Domain: $(-\infty, \infty)$

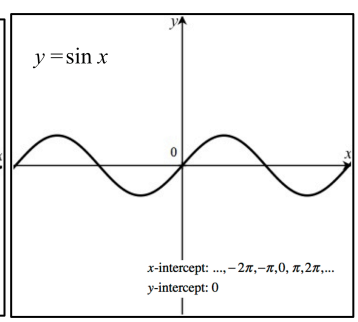
Range: $(0, \infty)$



Function: $y = e^{-x}$

Domain: $(-\infty, \infty)$

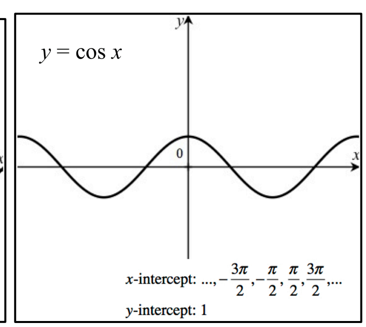
Range: $(0, \infty)$



Function: $y = \sin x$

Domain: $(-\infty, \infty)$

Range: $[-1, 1]$



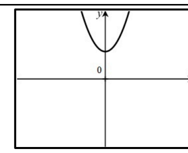
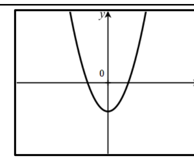
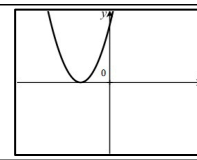
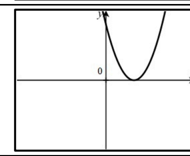
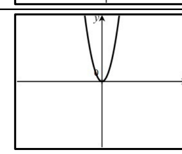
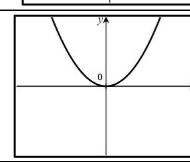
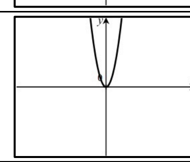
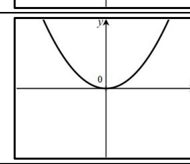
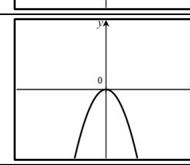
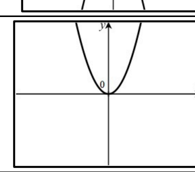
Function: $y = \cos x$

Domain: $(-\infty, \infty)$

Range: $[-1, 1]$

E. Transformation of Graphs

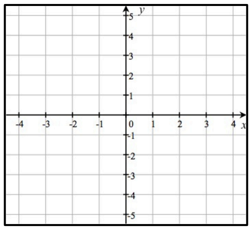
A curve in the form $y = f(x)$, which is one of the basic common functions from section C can be transformed in a variety of ways. The shape of the resulting curve stays the same but zeros and *y*-intercepts might change and the graph could be reversed. The table below describes transformations to a general function $y = f(x)$ with the parabolic function $f(x) = x^2$ as an example.

Notation	How $f(x)$ changes	Example with $f(x) = x^2$
$f(x)+a$	Moves graph up *a* units	
$f(x)-a$	Moves graph down *a* units	
$f(x+a)$	Moves graph *a* units left	
$f(x-a)$	Moves graph *a* units right	
$a \cdot f(x)$	$a > 1$: Vertical Stretch	
$a \cdot f(x)$	$0 < a < 1$: Vertical shrink	
$f(ax)$	$a > 1$: Horizontal compress (same effect as vertical stretch)	
$f(ax)$	$0 < a < 1$: Horizontal elongated (same effect as vertical shrink)	
$-f(x)$	Reflection across *x*-axis	
$f(-x)$	Reflection across *y*-axis	

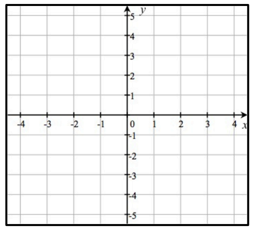
E. Transformation of Graphs Assignment

- Sketch the following equations:

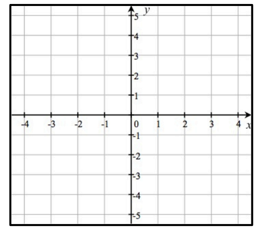
1. $y=-x^2$



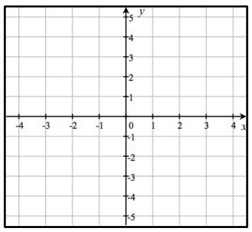
2. $y=2x^2$



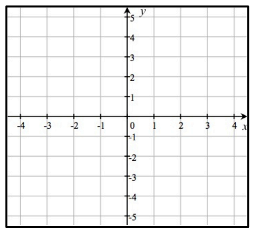
3. $y=(x-2)^2$



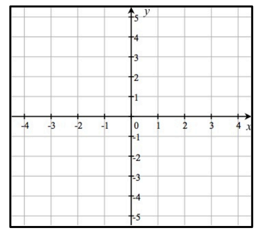
4. $y=2-\sqrt{x}$



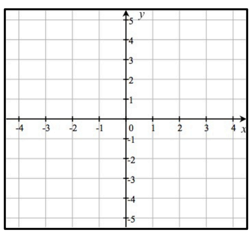
5. $y=\sqrt{x+1}+1$



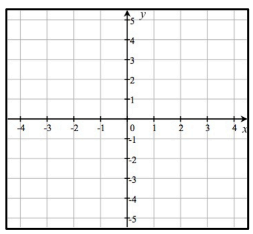
6. $y=\sqrt{4x}$



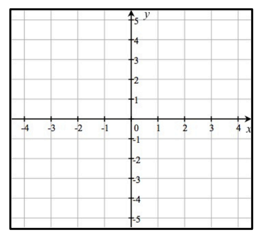
7. $y=|x+1|-3$



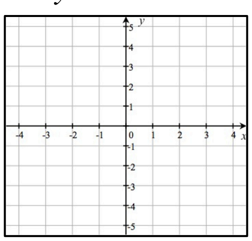
8. $y=-2|x-1|+4$



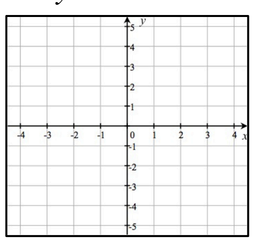
9. $y=-\left|\dfrac{x}{2}\right|-1$



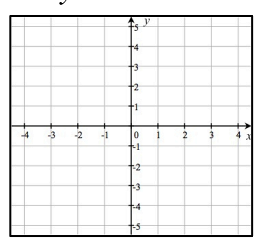
10. $y=2^x-2$



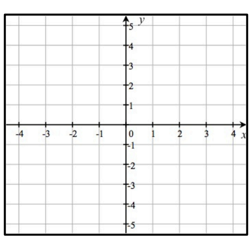
11. $y=-2^{x+2}$



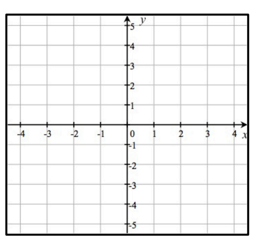
12. $y=2^{-2x}$



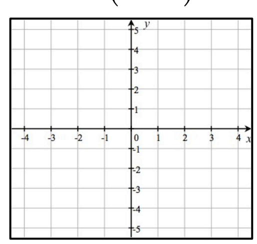
13. $y=\dfrac{1}{x-2}$



14. $y=\dfrac{-2}{x+1}$



15. $y=\dfrac{1}{(x+2)^2}-3$



F. Special Factorization

While factoring skills were more important in the days when A topics were specifically tested, students still must know how to factor. The special forms that occur most regularly are:

Common factor: $x^3+x^2+x=x\left(x^2+x+1\right)$

Difference of squares: $x^2-y^2=(x+y)(x-y)$ or $x^{2n}-y^{2n}=\left(x^n+y^n\right)\left(x^n-y^n\right)$

Perfect squares: $x^2+2xy+y^2=(x+y)^2$

Perfect squares: $x^2-2xy+y^2=(x-y)^2$

Sum of cubes: $x^3+y^3=(x+y)\left(x^2-xy+y^2\right)$ - Trinomial unfactorable

Difference of cubes: $x^3-y^3=(x-y)\left(x^2+xy+y^2\right)$ - Trinomial unfactorable

Grouping: $xy+xb+ay+ab=x(y+b)+a(y+b)=(x+a)(y+b)$

The term "factoring" usually means that coefficients are rational numbers. For instance, x^2-2 could technically be factored as $\left(x+\sqrt{2}\right)\left(x-\sqrt{2}\right)$ but since $\sqrt{2}$ is not rational, we say that x^2-2 is not factorable. It is important to know that x^2+y^2 is unfactorable.

• Completely factor the following expressions.

1. $4a^2+2a$
   $\boxed{2a(2a+1)}$

2. $x^2+16x+64$
   $\boxed{(x+8)^2}$

3. $4x^2-64$
   $\boxed{4(x+4)(x-4)}$

4. $\cancel{5x^4-5y^4}$
   $\cancel{\boxed{5\left(x^2+y^2\right)(x+y)(x-y)}}$

5. $16x^2-8x+1$
   $\boxed{(4x-1)^2}$

6. $9a^4-a^2b^2$
   $\boxed{a^2(3a+b)(3a-b)}$

7. $2x^2-40x+200$
   $\boxed{2(x-10)^2}$

8. x^3-8
   $\boxed{(x-2)\left(x^2+2x+4\right)}$

9. $8x^3+27y^3$
   $\boxed{(2x+3y)\left(4x^2-6xy+9y^2\right)}$

10. $\cancel{x^4-11x^2-80}$
   $\cancel{\boxed{(x+4)(x-4)\left(x^2+5\right)}}$

11. $\cancel{x^4-10x^2+9}$
   $\cancel{\boxed{(x+1)(x-1)(x+3)(x-3)}}$

12. $36x^2-64$
   $\boxed{4(3x+4)(3x-4)}$

13. x^3-x^2+3x-3
   $x^2(x-1)+3(x-1)$
   $(x-1)\left(x^2+3\right)$

14. $x^3+5x^2-4x-20$
   $x^2(x+5)-4(x+5)$
   $(x+5)(x-2)(x+2)$

15. $\cancel{9-\left(x^2+2xy+y^2\right)}$
   $\cancel{9-(x+y)^2}$
   $\cancel{(3+x+y)(3-x-y)}$

F. Special Factorization - Assignment

• Completely factor the following expressions

1. $x^3 - 25x$
2. $30x - 9x^2 - 25$
3. $3x^3 - 5x^2 + 2x$
4. $\cancel{3x^3 - 3}$
5. $16x^4 - 24x^2y + 9y^2$
6. $9a^4 - a^2b^2$
7. $\cancel{4x^4 + 7x^2 - 36}$
8. $250x^3 - 128$
9. $\cancel{\dfrac{8x^3}{125} + \dfrac{64}{y^3}}$
10. $x^5 + 17x^3 + 16x$
11. $144 + 32x^2 - x^4$
12. $16x^{4a} - y^{8a}$
13. $x^3 - xy^2 + x^2y - y^3$
14. $x^6 - 9x^4 - 81x^2 + 729$
15. $\cancel{x^2 - 8xy + 16y^2 - 25}$
16. $x^5 + x^3 + x^2 + 1$
17. $x^6 - 1$
18. $x^6 + 1$

G. Linear Functions

Probably the most important concept from precalculus that is required for differential calculus is that of linear functions. The formulas you need to know backwards and forwards are:

Slope: Given two points (x_1, y_1) and (x_2, y_2), the slope of the line passing through the points can be written as: $m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$.

Slope intercept form: the equation of a line with slope m and y-intercept b is given by $y = mx + b$.

Point-slope form: the equation of a line passing through the points (x_1, y_1) and slope m is given by $y - y_1 = m(x - x_1)$. While you might have preferred the simplicity of the $y = mx + b$ form in your algebra course, the $y - y_1 = m(x - x_1)$ form is far more useful in calculus.

Intercept form: the equation of a line with x-intercept a and y-intercept b is given by $\frac{x}{a} + \frac{y}{b} = 1$.

General form: $Ax + By + C = 0$ where A, B and C are integers. While your algebra teacher might have required your changing the equation $y - 1 = 2(x - 5)$ to general form $2x - y - 9 = 0$, you will find that on the AP calculus test, it is sufficient to leave equations for a lines in point-slope form and it is recommended not to waste time changing it unless you are specifically told to do so.

Parallel lines Two distinct lines are parallel if they have the same slope: $m_1 = m_2$.

Normal lines: Two lines are normal (perpendicular) if their slopes are negative reciprocals: $m_1 \cdot m_2 = -1$.

Horizontal lines have slope zero. **Vertical lines** have no slope (slope is undefined).

1. Find the equation of the line in slope-intercept form, with the given slope, passing through the given point.

a. $m = -4, (1,2)$

$\boxed{y - 2 = -4(x - 1) \Rightarrow y = -4x + 6}$

b. $m = \frac{2}{3}, (5,1)$

$\boxed{y - 1 = \frac{2}{3}(x - 5) \Rightarrow y = \frac{2x}{3} - \frac{7}{3}}$

c. $m = 0, \left(-\frac{1}{2}, \frac{3}{4}\right)$

$\boxed{y = -\frac{3}{4}}$

2. Find the equation of the line in slope-intercept form, passing through the following points.

a. $(4,5)$ and $(-2,-1)$

$$\boxed{\begin{array}{l} m = \frac{5+1}{4+2} = 1 \\ y - 5 = x - 4 \Rightarrow y = x + 1 \end{array}}$$

b. $(0,-3)$ and $(-5,3)$

$$\boxed{\begin{array}{l} m = \frac{3+3}{-5-0} = \frac{-6}{5} \\ y + 3 = \frac{-6}{5}x \Rightarrow y = \frac{-6}{5}x - 3 \end{array}}$$

c. $\left(\frac{3}{4}, -1\right)$ and $\left(1, \frac{1}{2}\right)$

$$\boxed{\begin{array}{l} m = \left(\frac{\frac{1}{2}+1}{1-\frac{3}{4}}\right)\left(\frac{4}{4}\right) = \frac{2+4}{4-3} = 6 \\ y - \frac{1}{2} = 6(x - 1) \Rightarrow y = 6x - \frac{11}{2} \end{array}}$$

3. Write equations of the line through the given point a) parallel and b) normal to the given line.

a. $(4,7)$, $4x - 2y = 1$

$$\boxed{\begin{array}{l} y = 2x - \frac{1}{2} \Rightarrow m = 2 \\ \text{a) } y - 7 = 2(x - 4) \qquad \text{b) } y - 7 = \frac{-1}{2}(x - 4) \end{array}}$$

b. $\left(\frac{2}{3}, 1\right)$, $x + 5y = 2$

$$\boxed{\begin{array}{l} y = \frac{-1}{5}x + 2 \Rightarrow m = \frac{-1}{5} \\ \text{a) } y - 1 = \frac{-1}{5}\left(x - \frac{2}{3}\right) \qquad \text{b) } y - 1 = 5\left(x - \frac{2}{3}\right) \end{array}}$$

G. Linear Functions - Assignment

1. Find the equation of the line in slope-intercept form, with the given slope, passing through the given point.

   a. $m=-7, (-3,-7)$ b. $m=\frac{-1}{2}, (2,-8)$ c. $m=\frac{2}{3}, \left(-6,\frac{1}{3}\right)$

2. Find the equation of the line in slope-intercept form, passing through the following points.

   a. $(-3,6)$ and $(-1,2)$ b. $(-7,1)$ and $(3,-4)$ c. $\left(-2,\frac{2}{3}\right)$ and $\left(\frac{1}{2},1\right)$

3. Write equations of the line through the given point a) parallel and b) normal to the given line.

   a. $(5,-3),\ x+y=4$ b. $(-6,2),\ 5x+2y=7$ c. $(-3,-4),\ y=-2$

4. Find an equation of the line containing $(4,-2)$ and parallel to the line containing $(-1,4)$ and $(2,3)$. Put your answer in general form.

5. Find k if the lines $3x-5y=9$ and $2x+ky=11$ are a) parallel and b) perpendicular.

H. Solving Quadratic Equations

Solving quadratics in the form of $ax^2+bx+c=0$ usually show up on the AP exam in the form of expressions that can easily be factored. But occasionally, you will be required to use the quadratic formula. When you have a quadratic equation, factor it, set each factor equal to zero and solve. If the quadratic equation doesn't factor or if factoring is too time-consuming, use the quadratic formula:

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$. The **discriminant** b^2-4ac will tell you how many solutions the quadratic has:

$$b^2-4ac\begin{cases} >0,\ \text{2 real solutions (if a perfect square, the solutions are rational)} \\ =0,\ \text{1 real solution} \\ <0,\ \text{0 real solutions (or 2 imaginary solutions, but AP calculus does not deal with imaginaries)} \end{cases}$$

1. Solve for x.

a. $x^2+3x+2=0$

$(x+2)(x+1)=0$
$x=-2, x=-1$

b. $x^2-10x+25=0$

$(x-5)^2=0$
$x=5$

c. $x^2-64=0$

$(x-8)(x+8)=0$
$x=8, x=-8$

d. $2x^2+9x=18$

$(2x-3)(x+6)=0$
$x=\dfrac{3}{2}, x=-6$

e. $12x^2+23x=-10$

$(4x+5)(3x+2)=0$
$x=-\dfrac{5}{4}, x=-\dfrac{2}{3}$

f. $48x-64x^2=9$

$(8x-3)^2=0$
$x=\dfrac{3}{8}$

g. $x^2+5x=2$

$x=\dfrac{-5\pm\sqrt{25+8}}{2}$
$x=\dfrac{-5\pm\sqrt{33}}{2}$

h. $8x-3x^2=2$

$x=\dfrac{8\pm\sqrt{64-24}}{6}$
$x=\dfrac{8\pm2\sqrt{10}}{6}=\dfrac{4\pm\sqrt{10}}{3}$

i. $6x^2+5x+3=0$

$x=\dfrac{-5\pm\sqrt{25-72}}{12}=\dfrac{-5\pm\sqrt{-47}}{12}$
No real solutions

j. $x^3-3x^2+3x-9=0$

$x^2(x-3)-3(x-3)=0$
$(x-3)(x^2-3)=0$
$x=3,\ x=\pm\sqrt{3}$

k. $\dfrac{x}{3}-\dfrac{5}{2}=\dfrac{-3}{x}$

$6x\left(\dfrac{x}{3}-\dfrac{5}{2}=\dfrac{-3}{x}\right)$
$2x^2-15x+18=0$
$(2x-3)(x-6)=0$
$x=\dfrac{3}{2},\ x=6$

l. $x^4-7x^2-8=0$

$(x^2-8)(x^2+1)=0$
$x=\pm\sqrt{8}=\pm2\sqrt{2}$

2. If $y=5x^2-3x+k$, for what values of k will the quadratic have two real solutions?

$(-3)^2-4(5)k>0\Rightarrow 9-20k>0\Rightarrow k<\dfrac{9}{20}$

H. Solving Quadratic Equations Assignment

1. Solve for x.

a. $x^2 + 7x - 18 = 0$

b. $x^2 + x + \frac{1}{4} = 0$

c. $2x^2 - 72 = 0$

d. $12x^2 - 5x = 2$

e. $20x^2 - 56x + 15 = 0$

f. $81x^2 + 72x + 16 = 0$

g. $x^2 + 10x = 7$

h. $3x - 4x^2 = -5$

i. $7x^2 - 7x + 2 = 0$

j. $x + \frac{1}{x} = \frac{17}{4}$

k. $x^3 - 5x^2 + 5x - 25 = 0$

l. $2x^4 - 15x^3 + 18x^2 = 0$

2. If $y = x^2 + kx - k$, for what values of k will the quadratic have two real solutions?

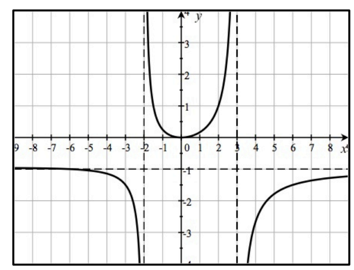
3. Find the domain of $y = \frac{2x-1}{6x^2 - 5x - 6}$.

I. Asymptotes

Rational functions in the form of $y=\dfrac{p(x)}{q(x)}$ possibly have vertical asymptotes, lines that the graph of the curve approach but never cross. To find the **vertical asymptotes**, factor out any common factors of numerator and denominator, reduce if possible, and then set the denominator equal to zero and solve.

Horizontal asymptotes are lines that the graph of the function approaches when x gets very large or very small. While you learn how to find these in calculus, a rule of thumb is that if the highest power of x is in the denominator, the horizontal asymptote is the line $y = 0$. If the highest power of x is both in numerator and denominator, the horizontal asymptote will be the line $y=\dfrac{\text{highest degree coefficient in numerator}}{\text{highest degree coefficient in denominator}}$. If the highest power of x is in the numerator, there is no horizontal asymptote, but a slant asymptote which is not used in calculus.

1) Find any vertical and horizontal asymptotes for the graph of $y=\dfrac{-x^2}{x^2-x-6}$.

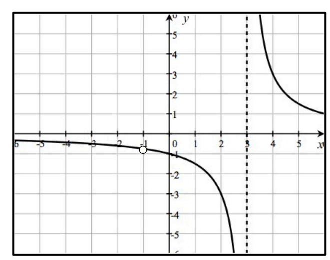


$$y=\frac{-x^2}{x^2-x-6}=\frac{-x^2}{(x-3)(x+2)}$$

Vertical asymptotes: $x-3=0\Rightarrow x=3$ and $x+2=0\Rightarrow x=-2$
Horizontal asymptotes: Since the highest power of x is 2 in both numerator and denominator, there is a horizontal asymptote at $y=-1$.

This is confirmed by the graph to the right. Note that the curve actually crosses its horizontal asymptote on the left side of the graph.

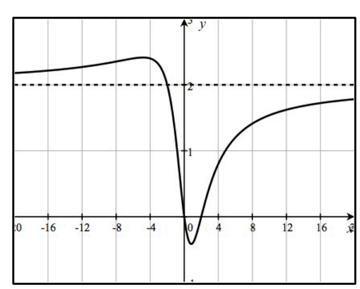
2) Find any vertical and horizontal asymptotes for the graph of $y=\dfrac{3x+3}{x^2-2x-3}$.



$$y=\frac{3x+3}{x^2-2x-3}=\frac{3(x+1)}{(x-3)(x+1)}=\frac{3}{x-3}$$

Vertical asymptotes: $x-3=0\Rightarrow x=3$. Note that since the $(x+1)$ cancels, there is no vertical asymptote at $x = 1$, but a hole (sometimes called a removable discontinuity) in the graph.
Horizontal asymptotes: Since there the highest power of x is in the denominator, there is a horizontal asymptote at $y = 0$ (the x-axis). This is confirmed by the graph to the right.

3) Find any vertical and horizontal asymptotes for the graph of $y=\dfrac{2x^2-4x}{x^2+4}$.



$$y=\frac{2x^2-4x}{x^2+4}=\frac{2x(x-2)}{x^2+4}$$

Vertical asymptotes: None. The denominator doesn't factor and setting it equal to zero has no solutions.
Horizontal asymptotes: Since the highest power of x is 2 in both numerator and denominator, there is a horizontal asymptote at $y = 2$. This is confirmed by the graph to the right.

I. Asymptotes - Assignment

• Find any vertical and horizontal asymptotes and if present, the location of holes, for the graph of

1. $y=\dfrac{x-1}{x+5}$

2. $y=\dfrac{8}{x^2}$

3. $y=\dfrac{2x+16}{x+8}$

4. $y=\dfrac{2x^2+6x}{x^2+5x+6}$

5. $y=\dfrac{x}{x^2-25}$

6. $y=\dfrac{x^2-5}{2x^2-12}$

7. $y=\dfrac{4+3x-x^2}{3x^2}$

8. $y=\dfrac{5x+1}{x^2-x-1}$

9. $y=\dfrac{1-x-5x^2}{x^2+x+1}$

10. $y=\dfrac{x^3}{x^2+4}$

11. $y=\dfrac{x^3+4x}{x^3-2x^2+4x-8}$

12. $y=\dfrac{10x+20}{x^3-2x^2-4x+8}$

13. $y=\dfrac{1}{x}-\dfrac{x}{x+2}$ (hint: express with a common denominator)

J. Negative and Fractional Exponents

In calculus, you will be required to perform algebraic manipulations with **negative exponents** as well as **fractional exponents**. You should know the definition of a negative exponent: $x^{-n} = \dfrac{1}{x^n}, x \neq 0$. Note that negative powers do not make expressions negative; they create fractions. Typically expressions in multiple-choice answers are written with positive exponents and students are required to eliminate negative exponents. Fractional exponents create roots. The definition of $x^{1/2} = \sqrt{x}$ and $x^{a/b} = \sqrt[b]{x^a} = \left(\sqrt[b]{x}\right)^a$.

As a reminder: when we multiply, we add exponents: $\left(x^a\right)\left(x^b\right) = x^{a+b}$.

When we divide, we subtract exponents: $\dfrac{x^a}{x^b} = x^{a-b}, x \neq 0$

When we raise powers, we multiply exponents: $\left(x^a\right)^b = x^{ab}$

In your algebra course, leaving an answer with a radical in the denominator was probably not allowed. You had to rationalize the denominator: $\dfrac{1}{\sqrt{x}}$ changed to $\left(\dfrac{1}{\sqrt{x}}\right)\left(\dfrac{\sqrt{x}}{\sqrt{x}}\right) = \dfrac{\sqrt{x}}{x}$. In calculus, you will find that it is not necessary to rationalize and it is recommended that you not take the time to do so.

• Simplify and write with positive exponents. Note: # 12 involves complex fractions, covered in section K.

1. $-8x^{-2}$

$\boxed{\dfrac{-8}{x^2}}$

2. $\left(-5x^3\right)^{-2}$

$\boxed{(-5)^{-2}x^{-6} = \dfrac{1}{(-5)^2 x^6} = \dfrac{1}{25x^6}}$

3. $\left(\dfrac{-3}{x^4}\right)^{-2}$

$\boxed{\dfrac{(-3)^{-2}}{\left(x^4\right)^{-2}} = \dfrac{1}{(-3)^2 x^{-8}} = \dfrac{x^8}{9}}$

4. $\left(36x^{10}\right)^{1/2}$

$\boxed{6x^5}$

5. $\left(27x^3\right)^{-2/3}$

$\boxed{\dfrac{1}{\left(27x^3\right)^{2/3}} = \dfrac{1}{9x^2}}$

6. $\left(16x^{-2}\right)^{3/4}$

$\boxed{16^{3/4}x^{-4/3} = \dfrac{8}{x^{4/3}}}$

7. $\left(x^{1/2} - x\right)^{-2}$

$\boxed{\dfrac{1}{\left(x^{1/2}-x\right)^2} = \dfrac{1}{x - 2x^{3/2} + x^2}}$

8. $\left(4x^2 - 12x + 9\right)^{-1/2}$

$\boxed{\dfrac{1}{\left[(2x-3)^2\right]^{1/2}} = \dfrac{1}{2x-3}}$

9. $\left(x^{1/3}\right)\left(\dfrac{1}{2}x^{-1/2}\right) + \left(x^{1/2}+1\right)\left(\dfrac{1}{3}x^{-1/3}\right)$

$\boxed{\dfrac{x^{1/3}}{2x^{1/2}} + \dfrac{x^{1/2}+1}{3x^{1/3}} = \dfrac{1}{2x^{1/6}} + \dfrac{x^{1/2}+1}{3x^{1/3}}}$

10. $\dfrac{-2}{3}(8x)^{-5/3}(8)$

$\boxed{\dfrac{-16}{3(8x)^{5/3}} = \dfrac{-16}{3(32)x^{5/3}} = -\dfrac{1}{6x^{5/3}}}$

11. $\dfrac{(x+4)^{1/2}}{(x-4)^{-1/2}}$

$\boxed{(x+4)^{1/2}(x-4)^{1/2} = \left(x^2-16\right)^{1/2}}$

12. $\left(x^{-1} + y^{-1}\right)^{-1}$

$\boxed{\left(\dfrac{1}{\dfrac{1}{x}+\dfrac{1}{y}}\right)\left(\dfrac{xy}{xy}\right) = \dfrac{xy}{y+x}}$

J. Negative and Fractional Exponents - Assignment

Simplify and write with positive exponents.

1. $-12^2 x^{-5}$

2. $\left(-12x^5\right)^{-2}$

3. $\left(4x^{-1}\right)^{-1}$

4. $\left(\dfrac{-4}{x^4}\right)^{-3}$

5. $\left(\dfrac{5x^3}{y^2}\right)^{-3}$

6. $\left(x^3-1\right)^{-2}$

7. $\left(121x^8\right)^{1/2}$

8. $\left(8x^2\right)^{-4/3}$

9. $\left(-32x^{-5}\right)^{-3/5}$

10. $(x+y)^{-2}$

11. $\left(x^3+3x^2+3x+1\right)^{-2/3}$

12. $x\left(x^{1/2}-x\right)^{-2}$

13. $\dfrac{1}{4}\left(16x^2\right)^{-3/4}(32x)$

14. $\dfrac{\left(x^2-1\right)^{-1/2}}{\left(x^2+1\right)^{1/2}}$

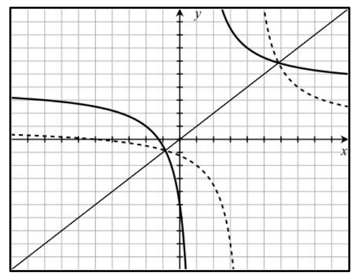
15. $\left(x^{-2}+2^{-2}\right)^{-1}$

L. Inverses

No topic in math confuses students more than inverses. If a function is a rule that maps x to y, an **inverse** is a rule that brings y back to the original x. If a point (x, y) is a point on a function f, then the point (y, x) is on the inverse function f^{-1}. Students mistakenly believe that since $x^{-1} = \frac{1}{x}$, then $f^{-1} = \frac{1}{f}$. This is decidedly incorrect. If a function is given in equation form, to find the inverse, replace all occurrences of x with y and all occurrences of y with x. If possible, then solve for y. Using the "horizontal-line test" on the original function f will quickly determine whether or not f^{-1} is also a function. By definition, $f\left(f^{-1}(x)\right) = x$. The domain of f^{-1} is the range of f and the range of f^{-1} is the domain of f.

1. Find the inverse to $y = \dfrac{4x+5}{x-1}$ and show graphically that its inverse is a function.

$$\boxed{\text{Inverse: } x = \frac{4y+5}{y-1} \Rightarrow xy - x = 4y + 5 \Rightarrow y = \frac{x+5}{x-4}}$$



Note that the function is drawn in bold and the inverse as dashed. The function and its inverse is symmetrical to the line $y = x$. The inverse is a function for two reasons: a) it passes the vertical line text or b) the function passes the horizontal line test.

2. Find the inverse to the following functions and show graphically that its inverse is a function.

a. $y = 4x - 3$

Inverse: $x = 4y - 3$
$y = \dfrac{x+3}{4}$ (function)

b. $y = x^2 + 1$

Inverse: $x = y^2 + 1$
$y = \pm\sqrt{x-1}$ (not a function)

c. $y = x^2 + 4x + 4$

Inverse: $x = y^2 + 4y + 4$
$x = (y+2)^2 \Rightarrow \pm\sqrt{x} = y + 2$
$y = -2 \pm \sqrt{x}$ (not a function)

3. Find the inverse to the following functions and show that $f\left(f^{-1}(x)\right) = x$

a. $f(x) = 7x + 4$

Inverse: $x = 7y + 4$
$y = f^{-1}(x) = \dfrac{x-4}{7}$
$f\left(\dfrac{x-4}{7}\right) = 7\left(\dfrac{x-4}{7}\right) + 4 = x$

b. $f(x) = \dfrac{1}{x-1}$

Inverse: $x = \dfrac{1}{y-1} \Rightarrow xy - x = 1$
$y = f^{-1}(x) = \dfrac{x+1}{x}$
$f\left(\dfrac{x+1}{x}\right) = \left(\dfrac{1}{\frac{x+1}{x} - 1}\right)\left(\dfrac{x}{x}\right)$
$= \dfrac{x}{x+1-x} = x$

c. $f(x) = x^3 - 1$

Inverse: $x = y^3 - 1$
$y = f^{-1}(x) = \sqrt[3]{x+1}$
$f\left(\sqrt[3]{x+1}\right) = \left(\sqrt[3]{x+1}\right)^3 - 1 = x$

4. Without finding the inverse, find the domain and range of the inverse to $f(x) = \sqrt{x+2} + 3$.

Function: Domain: $[-2, \infty)$, Range: $[3, \infty)$ Inverse: Domain: $[3, \infty)$, Range: $[-2, \infty)$

L. Inverses – Assignment

1. Find the inverse to the following functions and show graphically that its inverse is a function.

a. $2x-6y=1$ b. $y=ax+b$ c. $y=9-x^2$

d. $y=\sqrt{1-x^3}$ e. $y=\dfrac{9}{x}$ f. $y=\dfrac{2x+1}{3-2x}$

2. Find the inverse to the following functions and show that $f\left(f^{-1}(x)\right)=x$

a. $f(x)=\dfrac{1}{2}x-\dfrac{4}{5}$ b. $f(x)=x^2-4$ c. $f(x)=\dfrac{x^2}{x^2+1}$

3. Without finding the inverse, find the domain and range of the inverse to $f(x)=\dfrac{\sqrt{x+1}}{x^2}$.

P. Exponential Functions and Logarithms

Calculus spends a great deal of time on exponential functions in the form of b^x. Don't expect that when you start working with them in calculus, your teacher will review them. So learn them now! Students must know that the definition of a **logarithm** is based on exponential equations. If $y = b^x$ then $x = \log_b y$. So when you are trying to find the value of $\log_2 32$, state that $\log_2 32 = x$ and $2^x = 32$ and therefore $x = 5$.

If the base of a log statement is not specified, it is defined to be 10. When we asked for log 100, we are solving the equation: $10^x = 100$ and $x = 2$. The function $y = \log x$ has domain $(0,\infty)$ and range $(-\infty,\infty)$. In calculus, we primarily use logs with base e, which are called natural logs (ln). So finding ln 5 is the same as solving the equation $e^x = 5$. Students should know that the value of $e = 2.71828\ldots$

There are three rules that students must keep in mind that will simplify problems involving logs and natural logs. These rules work with logs of any base including natural logs.

i. $\log a + \log b = \log(a \cdot b)$ ii. $\log a - \log b = \log\left(\dfrac{a}{b}\right)$ iii. $\log a^b = b \log a$

1. Find a. $\log_4 8$

$$\boxed{\begin{array}{l} \log_4 8 = x \\ 4^x = 8 \Rightarrow 2^{2x} = 2^3 \\ x = \dfrac{3}{2} \end{array}}$$

b. $\ln\sqrt{e}$

$$\boxed{\begin{array}{l} \ln\sqrt{e} = x \\ e^x = e^{1/2} \\ x = \dfrac{1}{2} \end{array}}$$

c. $10^{\log 4}$

$$\boxed{\begin{array}{l} \log 4 = x \\ 10^x = 4 \text{ so } 10^{\log 4} = 4 \\ 10 \text{ to a power and log are inverses} \end{array}}$$

d. $\log 2 + \log 50$

$$\boxed{\begin{array}{l} \log(2 \cdot 50) = \log 100 \\ 2 \end{array}}$$

e. $\log_4 192 - \log_4 3$

$$\boxed{\begin{array}{l} \log_4\left(\dfrac{192}{3}\right) \\ \log_4 64 = 3 \end{array}}$$

f. $\ln\sqrt[5]{e^3}$

$$\boxed{\ln e^{3/5} = \dfrac{3}{5}\ln e = \dfrac{3}{5}}$$

2. Solve a. $\log_9\left(x^2 - x + 3\right) = \dfrac{1}{2}$

$$\boxed{\begin{array}{l} x^2 - x + 3 = 9^{1/2} \\ x(x-1) = 0 \\ x = 0, x = 1 \end{array}}$$

b. $\log_{36} x + \log_{36}(x-1) = \dfrac{1}{2}$

$$\boxed{\begin{array}{l} \log_{36} x(x-1) = \dfrac{1}{2} \\ x(x-1) = 36^{1/2} = 6 \\ x^2 - x - 6 = 0 \\ (x-3)(x+2) = 0 \\ \text{Only } x = 3 \text{ is in the domain} \end{array}}$$

c. $\ln x - \ln(x-1) = 1$

$$\boxed{\begin{array}{l} \ln\left(\dfrac{x}{x-1}\right) = 1 \\ \dfrac{x}{x-1} = e \Rightarrow x = ex - e \\ x = \dfrac{e}{e-1} \end{array}}$$

d. $5^x = 20$

$$\boxed{\begin{array}{l} \log\left(5^x\right) = \log 20 \\ x \log 5 = \log 20 \\ x = \dfrac{\log 20}{\log 5} \text{ or } x = \dfrac{\ln 20}{\ln 5} \end{array}}$$

e. $e^{-2x} = 5$

$$\boxed{\begin{array}{l} \ln e^{-2x} = \ln 5 \\ -2x = \ln 5 \Rightarrow x = \dfrac{-\ln 5}{2} \end{array}}$$

f. $2^x = 3^{x-1}$

$$\boxed{\begin{array}{l} \log\left(2^x\right) = \log\left(3^{x-1}\right) \\ x \log 2 = (x-1)\log 3 \\ x \log 2 = x \log 3 - \log 3 \Rightarrow x = \dfrac{\log 3}{\log 3 - \log 2} \end{array}}$$

P. Exponential Functions and Logarithms - Assignment

1. Find a. $\log_2 \frac{1}{4}$ b. $\log_8 4$ c. $\ln \frac{1}{\sqrt[3]{e^2}}$

d. $5^{\log_5 40}$ e. $e^{\ln 12}$ f. $\log_{12} 2 + \log_{12} 9 + \log_{12} 8$

g. $\log_2 \frac{2}{3} + \log_2 \frac{3}{32}$ h. $\log_{\frac{1}{3}} \frac{4}{3} - \log_{\frac{1}{3}} 12$ i. $\log_3 \left(\sqrt{3}\right)^5$

2. Solve a. $\log_5(3x-8)=2$ b. $\log_9\left(x^2-x+3\right)=\frac{1}{2}$ c. $\log(x-3)+\log 5=2$

d. $\log_2(x-1)+\log_2(x+3)=5$ e. $\log_5(x+3)-\log_5 x=2$ f. $\ln x^3-\ln x^2=\frac{1}{2}$

g. $3^{x-2}=18$ h. $e^{3x+1}=10$ i. $8^x=5^{2x-1}$

U. Graphical Solutions to Equations and Inequalities – Assignment

• Solve these equations or inequalities graphically.

1. $3x^3 - x - 5 = 0$

2. $x^3 - 5x^2 + 4x - 1 = 0$

3. $2x^2 - 1 = 2^x$

4. $2\ln(x+1) = 5\cos x$ on $[0, 2\pi)$

5. $x^4 - 9x^2 - 3x - 15 < 0$

6. $\dfrac{x^2 - 4x - 4}{x^2 + 1} > 0$ on $[0, 8]$

7. $x \sin x^2 > 0$ on $[0, 3]$

8. $\cos^{-1} x > x^2$ on $[-1, 1]$