

Math Review and Summer Work 2021

- I. Physics, and AP Physics in particular, requires a proficiency in algebra, trigonometry, and geometry. In addition to the science concepts, Physics often seems like a course in applied mathematics. The following assignment includes mathematical problems that are considered routine in AP Physics. This includes knowing several key metric system conversion factors and how to employ them. Understanding vectors is another key mathematical area used in Physics. You may recognize some part 1 materials from Honors Physical Science.
- II. The attached pages contain both math review and links to introductory physics tutorials and a summer problem set (on p4) of 34 problems on vectors and motion. It is hoped that these tutorials and problems, (combined with your previous math and science knowledge), will give you a head start to very rigorous AP Physics course.
- III. What if I don't get all the problems or don't understand the instructions?
- Do the best you can, but show some work / effort in order to receive credit.
 - You can communicate with me (Mr. Beck) via email: dbeck@fenwickfalcons.org
 - Come to class with your questions ready, in order to resolve these issues.
1. The following are ordinary physics formulas, with numbers in place of the variables normally seen. Typically, we need to round an answer to an appropriate number of significant figures and put the answer in scientific notation (when appropriate), and then simplify the units. Try to do that for these expressions:

(calculated result, rounded, with units)

a. $T_s = 2\pi \sqrt{\frac{4.5 \times 10^{-2} \text{ kg}}{2.0 \times 10^3 \text{ kg/s}^2}} =$ _____

b. $K = \frac{1}{2} (6.6 \times 10^2 \text{ kg}) (2.11 \times 10^4 \text{ m/s})^2 =$ _____

c. $F = \left(9.0 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \frac{(3.2 \times 10^{-9} \text{ C})(9.6 \times 10^{-9} \text{ C})}{(0.32 \text{ m})^2} =$ _____

d. $\frac{1}{R_p} = \frac{1}{4.5 \times 10^2 \Omega} + \frac{1}{9.4 \times 10^2 \Omega}$ $R_p =$ _____

e. $e = \frac{1.7 \times 10^3 \text{ J} - 3.3 \times 10^2 \text{ J}}{1.7 \times 10^3 \text{ J}} =$ _____

f. $1.33 \sin 25.0^\circ = 1.50 \sin \theta$ $\theta =$ _____

g. $K_{\text{max}} = (6.63 \times 10^{-34} \text{ J} \cdot \text{s}) (7.09 \times 10^{14} \text{ s}) - 2.17 \times 10^{-19} \text{ J} \cdot \text{s}^2 =$ _____

h. $\gamma = \frac{1}{\sqrt{1 - \frac{2.25 \times 10^8 \text{ m/s}}{3.00 \times 10^8 \text{ m/s}}}} =$ _____

Often problems on the AP exam are done with variables only. Solve for the variable indicated.

- i. $v^2 = v_o^2 + 2a(s - s_o)$, $a =$ _____
- j. $K = \frac{1}{2}kx^2$, $x =$ _____
- k. $T_p = 2\pi\sqrt{\frac{\ell}{g}}$, $g =$ _____
- l. $F_g = G\frac{m_1m_2}{r^2}$, $r =$ _____
- m. $mgh = \frac{1}{2}mv^2$, $v =$ _____
- n. $x = x_o + v_o t + \frac{1}{2}at^2$, $t =$ _____
- o. $B = \frac{\mu_o I}{2\pi r}$, $r =$ _____
- p. $x_m = \frac{m\lambda L}{d}$, $d =$ _____
- q. $pV = nRT$, $T =$ _____
- r. $\sin\theta_c = \frac{n_1}{n_2}$, $\theta_c =$ _____
- s. $qV = \frac{1}{2}mv^2$, $v =$ _____
- t. $\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i}$, $s_i =$ _____

2. Physics uses the **MKS** system (**SI**: System Internationale). **MKS** stands for meter, kilogram, second. These are the usual units of choice of physics. The equations in physics depend on unit agreement, so you must convert to **MKS** in most problems to arrive at the correct answer. Sometimes you may need to convert to or from alternative systems, such as the **CGS** version (centimeter, gram, second) of SI still popular in the United Kingdom. For example, in MKS SI, energy is measured in Joules, where $1 \text{ J} = 1 \text{ kg m}^2/\text{s}^2$. In **CGS** SI energy is measured in ergs, where $1 \text{ erg} = 1 \text{ g cm}^2/\text{s}^2$. Sometimes it is convenient to use still different units of energy, such as calories, where 1 calorie is defined as the amount of energy to raise 1 g of water by 1 Celsius degree in temperature. Despite the occasional exception, most of the work we do will use **MKS** SI units.

kilometers (*km*) to meters (*m*) and meters to kilometers
centimeters (*cm*) to meters (*m*) and meters to centimeters
millimeters (*mm*) to meters (*m*) and meters to millimeters
nanometers (*nm*) to meters (*m*) and meters to nanometers
micrometers (μm) to meters (*m*)

gram (*g*) to kilogram (*kg*)
Celsius ($^{\circ}\text{C}$) to Kelvin (*K*)
atmospheres (*atm*) to Pascals (*Pa*)
liters (*L*) to cubic meters (m^3)

Other conversions will be taught as they become necessary.

3. What if you don't know the conversion factors? Colleges want students who can find their own information (so do employers). Hint: Try a good dictionary and look under "measure" or "measurement". Or the Internet;-) Some smartphone apps exist to solve this common problem, too, but you won't have access to them on exams. I usually put common conversion factors on a formula sheet I pass out with quizzes and tests. Practice the following conversions, looking up ones you don't already know:

- a. 4008 g = _____ *kg*
b. 1.2 km = _____ *m*
c. 823 nm = _____ *m*
d. 298 K = _____ $^{\circ}\text{C}$
e. 0.77 m = _____ *cm*
f. $8.8 \times 10^{-8} \text{ m}$ = _____ *mm*
g. 1.2 atm = _____ *Pa*

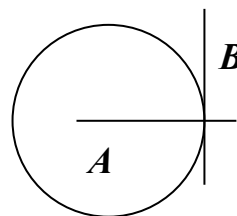
- h. $25.0 \mu\text{m}$ = _____ *m*
i. 2.65 mm = _____ *m*
j. 8.23 m = _____ *km*
k. 5.4 L = _____ m^3
l. 40.0 cm = _____ *m*
m. $6.23 \times 10^{-7} \text{ m}$ = _____ *nm*
n. $1.5 \times 10^{11} \text{ m}$ = _____ *km*

4. Solve the following geometric problems.

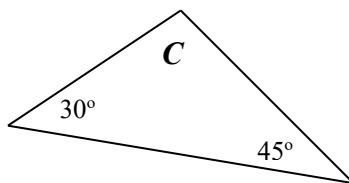
a. Line **B** touches the circle at a single point. Line **A** extends through the center of the circle.

i. What is line **B** in reference to the circle?

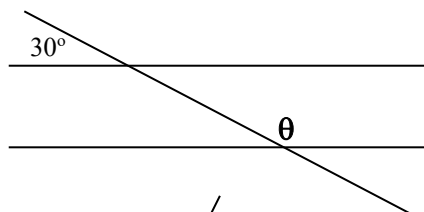
ii. How large is the angle between lines **A** and **B**?



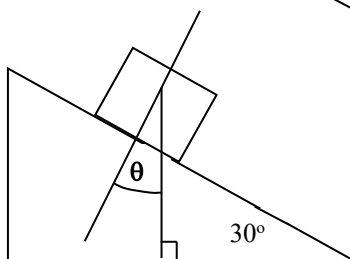
b. What is angle **C**?



c. What is angle θ ?



d. How large is θ ?

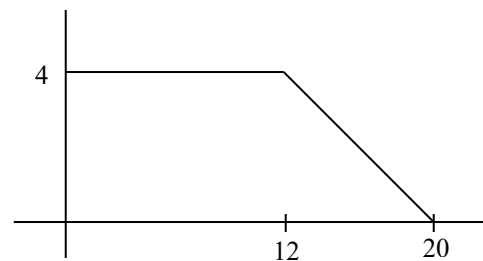


e. The radius of a circle is 5.5 cm,

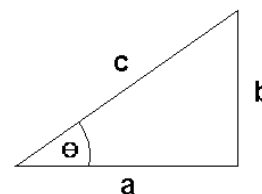
i. What is the circumference in meters?

ii. What is its area in square meters?

f. What is the area under the curve at the right?



7. Using the generic triangle to the right, Right Triangle Trigonometry and Pythagorean Theorem solve the following. **Your calculator must be in degree mode.** Round your answers to 2 sig figs and include units.



- | | |
|--|--|
| a. $\theta = 55^\circ$ and $c = 32\text{ m}$, solve for a and b . | d. $a = 250\text{ m}$ and $b = 180\text{ m}$, solve for θ and c . |
| b. $\theta = 45^\circ$ and $a = 15\text{ m/s}$, solve for b and c . | e. $a = 25\text{ cm}$ and $c = 32\text{ cm}$, solve for b and θ . |
| c. $b = 17.8\text{ m}$ and $\theta = 65^\circ$, solve for a and c . | f. $b = 65\text{ cm}$ and $c = 104\text{ cm}$, solve for a and θ . |

Vectors

Most of the quantities in physics are vectors. **This makes proficiency in vectors extremely important.**

Open the tutorial on vectors at the website: <http://www.physicsclassroom.com/Class/vectors/>

Read the tutorial and complete the worksheets attached to the rest of this packet.

Lesson 1: Vectors - Fundamentals and Operations

- [Vectors and Direction](#)
- [Vector Addition](#)
- [Resultants](#)
- [Vector Components](#)
- [Vector Resolution](#)
- [Component Method of Vector Addition](#)
- [Relative Velocity and Riverboat Problems](#)
- [Independence of Perpendicular Components of Motion](#)

1D Kinematics Practice Problems

To get a head start beyond vector fundamentals, to solve actual problems where vectors must be used, read the following tutorials and try the **Summer Problem Set Assignment** mentioned below:

Lesson 1 : Describing Motion with Words

- [Introduction to the Language of Kinematics](#)
- [Scalars and Vectors](#)
- [Distance and Displacement](#)
- [Speed and Velocity](#)
- [Acceleration](#)

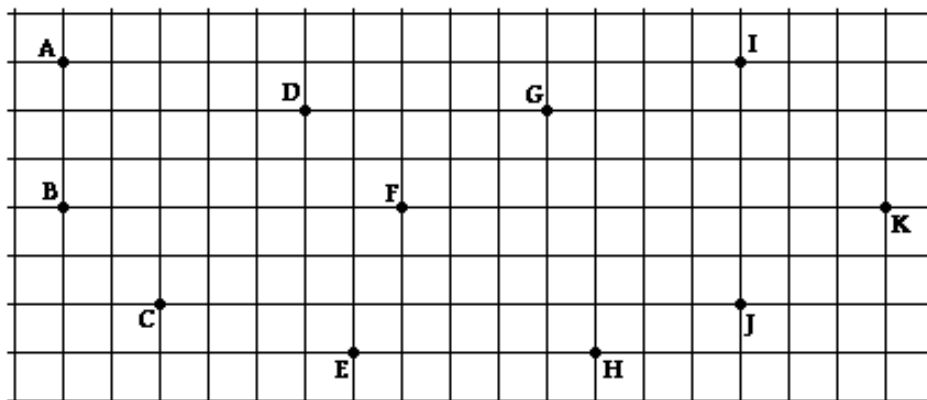
Summer Problem Set Assignment: (34 problems, due the 2nd class meeting of the year)

Go to <http://www.physicsclassroom.com/calcpad/vecproj/problems> and complete the 34 problems on this page, showing your handwritten, neatly organized work on sheets of notebook paper. Each problem can be checked while online for a correct answer and an audio guide to the solution, if further help is needed. Show all steps of your solution, not just answers!!

The remaining pages are to be completed based on the "Lesson 1 : Vectors" links mentioned above:

Vectors and Projectiles

8. Consider the grid below with several marked locations.



Determine the direction of the resultant displacement for a person who walks from location ...

- a. A to C: _____ b. D to B: _____ c. G to D: _____
d. F to A: _____ e. F to E: _____ f. C to H: _____
g. E to K: _____ h. J to K to F: _____ i. I to K to B: _____

9. A short verbal description of a vector quantity is given in each of the descriptions below. Read the description, select a scale, draw a set of axes, and construct a scaled vector diagram to represent the given vector quantity.

a. Kent Holditnomore excused himself from class, grabbed the cardboard pass off the lecture table, and displaced himself 10 meters at 170° .

b. Marcus Tardee took an extended lunch break and found himself hurrying through the hallways to physics class. After checking in at the attendance office, Marcus moved with an average velocity of 5.0 m/s at 305° .

Addition of Vectors

Read from **Lesson 1** of the **Vectors and Motion in Two-Dimensions** chapter at **The Physics Classroom**:

<http://www.physicsclassroom.com/Class/vectors/u3l1b.html>

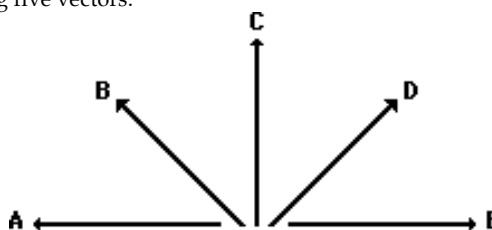
<http://www.physicsclassroom.com/Class/vectors/u3l1c.html>

MOP Connection: Vectors and Projectiles: sublevels 2, 3 and 4

1. Aaron Agin recently submitted his vector addition homework. As seen below, Aaron added two vectors and drew the resultant. However, Aaron Agin failed to label the resultant on the diagram. For each case, identify the resultant (A, B, or C). Finally, indicate what two vectors Aaron added to achieve this resultant (express as an equation such as $X + Y = Z$) and approximate the direction of the resultant.

	<p>Resultant is: _____</p> <p>Vector Eq'n: _____</p> <p>Dir'n of R: _____</p>
	<p>Resultant is: _____</p> <p>Vector Eq'n: _____</p> <p>Dir'n of R: _____</p>
	<p>Resultant is: _____</p> <p>Vector Eq'n: _____</p> <p>Dir'n of R: _____</p>

2. Consider the following five vectors.



Sketch the following and draw the resultant (R). Do not draw a scaled vector diagram; merely make a sketch. Label each vector. Clearly label the resultant (R).

$$A + B + D$$

$$A + C + D$$

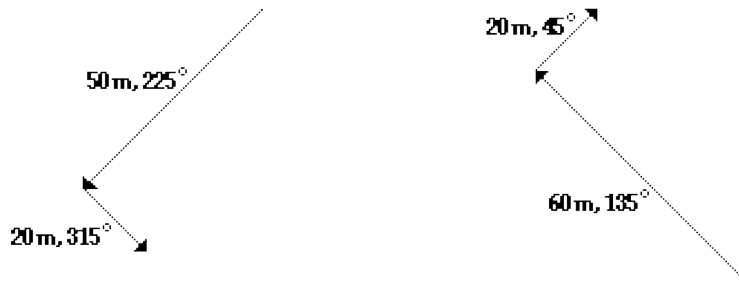
$$B + C + E$$

Vectors and Projectiles

Math Skill:

Vectors that make right angles to each other can be added together using Pythagorean theorem. Use Pythagorean theorem to solve the following problems.

3. While Dexter is on a camping trip with his Boy Scout troop, the scout leader gives each boy a compass and a map. Dexter's map contains several sets of directions. For the two sets below, draw and label the resultant (**R**). Then use the Pythagorean theorem to determine the magnitude of the resultant displacement for each set of two directions. **PSYW**
- Dexter walked 50 meters at a direction of 225° and then walked 20 meters at a direction of 315° .
 - Dexter walked 60 meters at a direction of 135° and then walked 20 meters at a direction of 45° .



4. In a classroom lab, a Physics student walks through the hallways making several small displacements to result in a single overall displacement. The listings below show the individual displacements for students A and B. Simplify the collection of displacements into a pair of N-S and E-W displacements. Then use Pythagorean theorem to determine the overall displacement.

Student A	Student B
2 m, North	2 m, North
16 m, East	12 m, West
14 m, South	14 m, South
2 m, West	56 m, West
12 m, South	12 m, South
46 m, West	36 m, East
$\Sigma \text{ E-W} =$ _____	$\Sigma \text{ E-W} =$ _____
$\Sigma \text{ N-S} =$ _____	$\Sigma \text{ N-S} =$ _____
Overall Displacement:	Overall Displacement:

Vector Components, Vector Resolution and Vector Addition

Read from **Lesson 1** of the **Vectors and Motion in Two-Dimensions** chapter at **The Physics Classroom**:

<http://www.physicsclassroom.com/Class/vectors/u3l1b.html>

<http://www.physicsclassroom.com/Class/vectors/u3l1c.html>

<http://www.physicsclassroom.com/Class/vectors/u3l1eb.cfm>

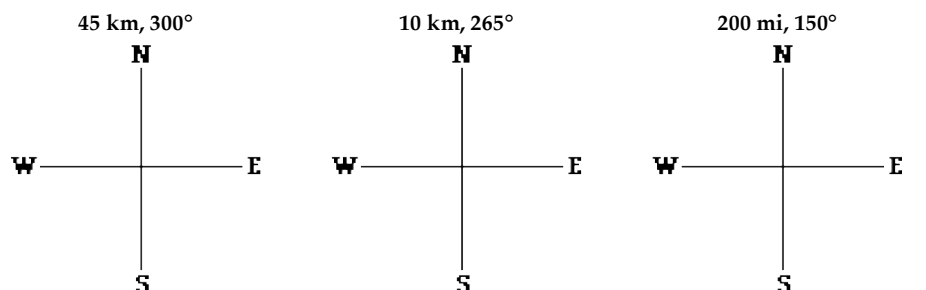
MOP Connection: Vectors and Projectiles: sublevels 3 and 5

Review: The direction of a vector is often expressed as a counter-clockwise (CCW) angle of rotation of that vector from due east (i.e., the horizontal). In such a convention, East is 0° , North is 90° , West is 180° and South is 270° .

About Vector Components:

A vector directed at 120° CCW has a direction which is a little west and a little more north. Such a vector is said to have a northward and a westward component. A **component** is simply the effect of the vector in a given direction. A hiker with a 120° displacement vector is displaced both northward and westward; there are two separate effects of such a displacement upon the hiker.

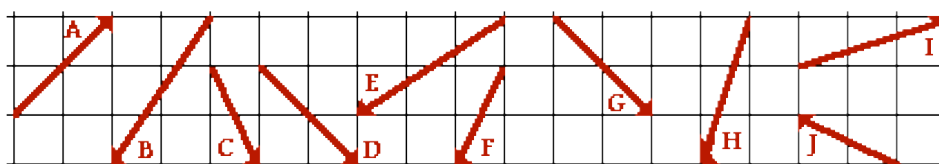
- Sketch the given vectors; determine the direction of the two components by circling two directions (N, S, E or W). Finally indicate which component (or effect) is greatest in magnitude.



Components: E W N S Components: E W N S Components: E W N S

Greatest magnitude? _____ Greatest magnitude? _____ Greatest magnitude? _____

- Consider the various vectors below. Given that each square is 10 km along its edge, determine the magnitude and direction of the components of these vectors.




Vector	E-W Component (mag. & dirn')	N-S Component (mag. & dirn')
A		
C		
E		
G		
I		

Vector	E-W Component (mag. & dirn')	N-S Component (mag. & dirn')
B		
D		
F		
H		
J		

Vectors and Projectiles

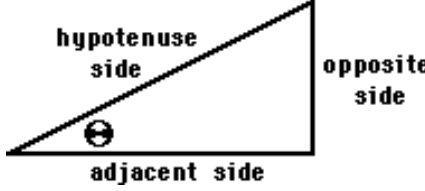
The magnitude of a vector component can be determined using trigonometric functions.



TIP
Trigonometry
Review

Trigonometric functions are mathematical functions that relate the length of the sides of a right triangle to the angles of the triangle. The meaning of the functions can be easily remembered by the mnemonic

SOH CAH TOA

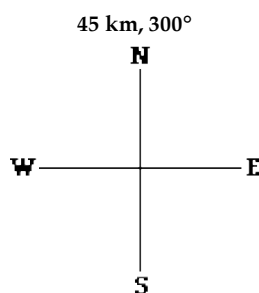


SOH --> $\sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$

CAH --> $\cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$

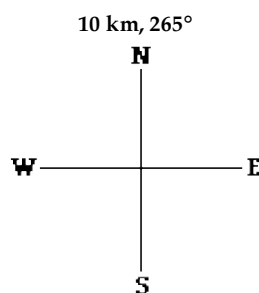
TOA --> $\tan \theta = \frac{\text{Opposite}}{\text{Adjacent}}$

3. Sketch the given vectors; project the vector onto the coordinate axes and sketch the components. Then determine the magnitude of the components using SOH CAH TOA.



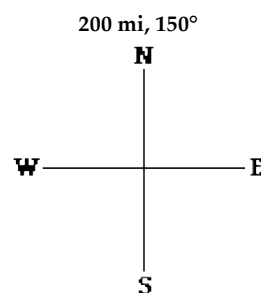
E-W Component:

N-S Component:



E-W Component:

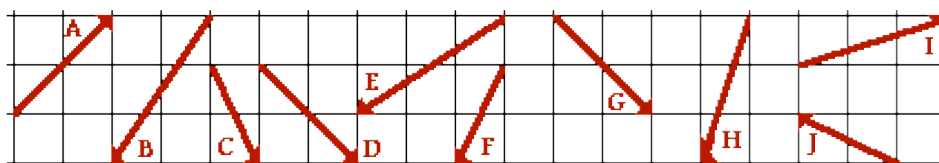
N-S Component:



E-W Component:

N-S Component:

4. Consider the diagram below (again); each square is 10 km along its edge. Use components and vector addition to determine the resultant displacement (magnitude only) of the following:





A + B + C ==> Σ E-W: _____	Σ N-S: _____	Overall Displacement: _____
D + E + F ==> Σ E-W: _____	Σ N-S: _____	Overall Displacement: _____
G + H + I ==> Σ E-W: _____	Σ N-S: _____	Overall Displacement: _____
A + J + G ==> Σ E-W: _____	Σ N-S: _____	Overall Displacement: _____

Vector Addition by Components

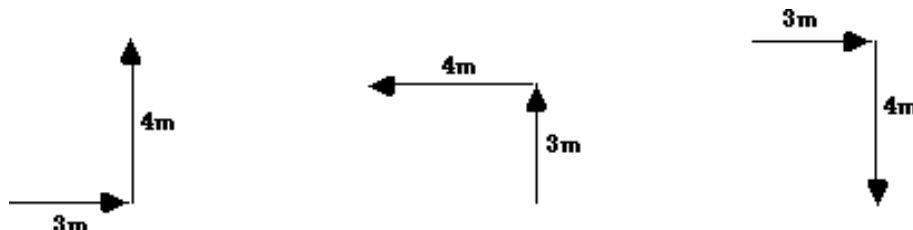
Read from **Lesson 1** of the **Vectors and Motion in Two-Dimensions** chapter at The Physics Classroom:

<http://www.physicsclassroom.com/Class/vectors/u3l1eb.cfm>

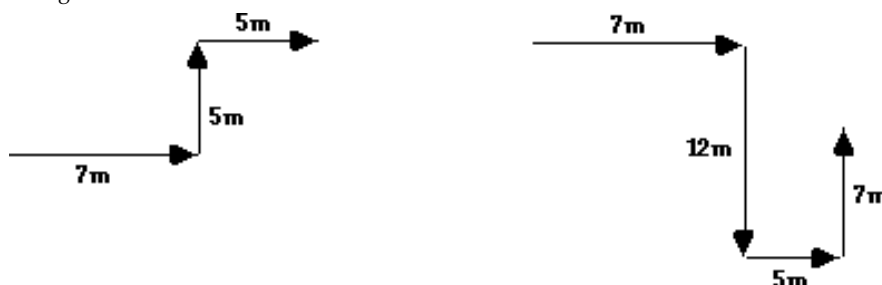
MOP Connection: Vectors and Projectiles: sublevels 3 and 4

 TIP Trigonometry Review	Trigonometric functions are mathematical functions that relate the length of the sides of a right triangle to the angles of the triangle. The meaning of the functions can be easily remembered by the mnemonic <p style="text-align: center;">SOH CAH TOA</p>	
<div style="display: flex; justify-content: space-between;"> <div>SOH --> $\sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$</div> <div>CAH --> $\cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$</div> <div>TOA --> $\tan \theta = \frac{\text{Opposite}}{\text{Adjacent}}$</div> </div>		

1. For the following vector addition diagrams, use Pythagorean Theorem to determine the magnitude of the resultant. Use SOH CAH TOA to determine the direction. **PSAYW**

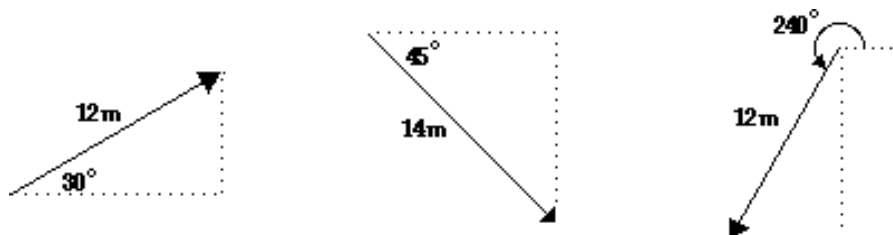


2. Use the Pythagorean Theorem and SOH CAH TOA to determine the magnitude and direction of the following resultants.

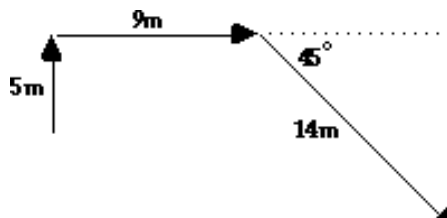
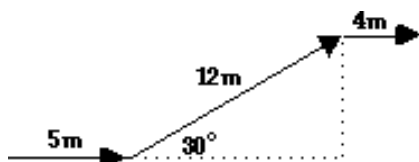


Vectors and Projectiles

3. A component is the effect of a vector in a given x- or y- direction. A component can be thought of as the projection of a vector onto the nearest x- or y-axis. SOH CAH TOA allows a student to determine a component from the magnitude and direction of a vector. Determine the components of the following vectors.



4. Consider the following vector diagrams for the displacement of a hiker. For any *angled* vector, use SOH CAH TOA to determine the components. Then sketch the resultant and determine the magnitude and direction of the resultant.



Vector Representation

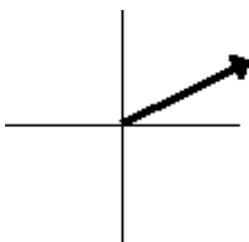
Read from **Lesson 1** of the **Vectors and Motion in Two-Dimensions** chapter at **The Physics Classroom**:

<http://www.physicsclassroom.com/Class/vectors/u3l1a.html>

MOP Connection: Vectors and Projectiles: sublevel 1

Vector quantities are quantities that have both magnitude and direction. The direction of a vector is often expressed as a counter-clockwise angle of rotation of that vector from due east (i.e., the horizontal). For questions #1-6, indicate the direction of the following vectors.

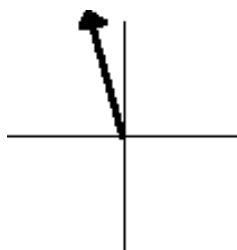
1.



CCW Dir'n: _____

magnitude: _____

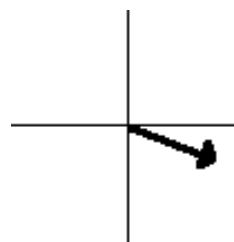
2.



CCW Dir'n: _____

magnitude: _____

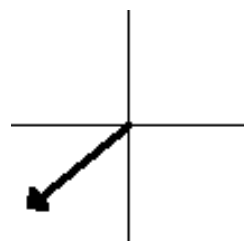
3.



CCW Dir'n: _____

magnitude: _____

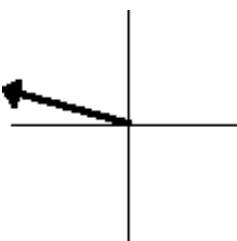
4.



CCW Dir'n: _____

magnitude: _____

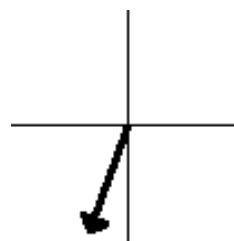
5.



CCW Dir'n: _____

magnitude: _____

6.



CCW Dir'n: _____

magnitude: _____

7. The above diagrams are referred to as scaled vector diagrams. In a scaled vector diagram, the magnitude of a vector is represented by its length. A scale is used to convert the length of the arrow to the magnitude of the vector quantity. Determine the magnitude of the above six vectors if given the scale: 1 cm = 10 m/s.